Statistical distribution of IPHC age readings

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Abstract

This paper reports estimates of the statistical distributions of surface and break-and-burn readings about the true age of a fish (assuming that break-and-burn readings are on average correct). The variance of single readings of a given otolith is low for both types. Surface readings are increasingly biased downward after age 12, and among older fish they have a large variance due to variation among otoliths of a given age in the number of annuli countable by the surface method. An Appendix contains detailed instructions for smearing age distributions.

Introduction

Age readers strive to follow consistent rules when counting annuli on an otolith, but there is still some judgment involved and consequently some variability both within and among readers in the age assigned to a given otolith. A single age reading can therefore be regarded as a draw from a probability distribution. An estimate of the distribution can be incorporated into the stock assessment model to predict the observed distribution of age readings that would result from an underlying true age composition, and thereby sharpen estimates of abundance and especially year-class strengths.

In concept, the distribution of readings of a single otolith is what would be observed if the same otolith were read many times by experienced readers following the same protocol. The mode of this distribution is by definition the correct age to assign to the otolith according to that protocol, whether or not it is the true age of the fish. For clarity this modal age will be called the "canonical age" of an otolith.

At the International Pacific Halibut Commission (IPHC), otoliths were aged by surface reading until the early 1990s. During the 1990s an increasing number of difficult and older otoliths were broken and burned for reading because it was known that surface readings tended to be too low in those cases, but surface readings were continued for a majority of fish until 2002, when surface reading was discontinued altogether in favor of breaking and burning.

Break-and-burn readings have recently been validated by comparing them with a reference chronology of ^{14}C uptake resulting from nuclear tests in the mid-20th century (Piner and Wischniowski in review), so we now know that the break-and-burn protocol is accurate; i.e. that the canonical break-and-burn age of an otolith is the true age of the fish, and that the canonical surface age is too low, at least for older fish. The problem is therefore to estimate the distribution of both surface and break-and-burn readings at each canonical break-and-burn age.

Distribution of break-and-burn readings about the canonical age

Given a sufficiently large sample of paired readings, the distribution of deviations of single readings from the canonical age can be estimated by fitting the sample distribution of differences between paired readings (Clark in review). This procedure shows that the unsigned deviations of break-and-burn readings follow a geometric distribution. To be specific, let the random variable b denote a single reading of an otolith of canonical age B, and let $v = b - B$ be the signed deviation, so v has the same distribution as b except that it is shifted so as to have a mode at zero. Then the probability of observing a given unsigned deviation $|v| = 0,1,2,\ldots$ is $f(|v|) = p \cdot q^{|v|}$ where $q = 1 - p$. The distribution is assumed to be symmetric, so the probability of observing a given *signed* deviation v is $f(v=0) = p$ and $f(v \neq 0) = p \cdot q^{|v|} / 2$. The single parameter $p = f(0)$ decreases with age as the variance of readings increases, and it can be computed from the sample variance σ_v^2 of the readings at each age with the formula $p = \frac{-3 + \sqrt{1 + 8 \cdot \sigma_v^2}}{2 \cdot (\sigma_v^2 - 1)}$. Otoliths can and must be grouped by mean assigned age for this purpose because the canonical age of an otolith is unknown. The variance of the signed deviations σ_v^2 can be estimated conveniently as half the variance of the signed difference between paired readings.

The standard deviation of break-and-burn readings increases with canonical age in a non-linear fashion (Fig. 1), apparently leveling off at some point. The shape of this curve at ages beyond 25 or so is not well determined by the amount of data presently available; it should be re-estimated in the future. For the time being, the trend is well-described by the fitted curve, which is:

$$\sigma_v(B) = 1.28 \cdot \left(1 - \exp\left(-0.100 \cdot (B - 2.93)\right)\right)$$

where *B* is the canonical break-and-burn age and by hypothesis the true age.

Parenthetically, the unsigned deviations of surface ages also follow a geometric distribution, and the standard deviation of surface age readings increases linearly with canonical *surface* age:

$$\sigma_v(A) = -0.112 + 0.0668 \cdot A$$

where *A* is canonical *surface* age, which for fish older than 12 or so is less than the canonical break-and-burn age. When corrected for the low bias of surface ages among older fish, this equation produces values similar to the standard deviations of break-and-burn readings for fish of the same true age: around 0.5 y at age 10 increasing to about 1 y at age 20 and continuing to increase thereafter. So for both kinds of reading the variance about the canonical age is modest.

Mean of surface readings at a given break-and-burn age

Through about age 12 there is little difference on average between surface and break-and-burn readings. (In fact surface ages are on average slightly higher for younger fish, but the difference is negligible.) Beyond a break-and-burn age of 12, however, surface readings are lower on average, by a growing margin (Fig. 2). The relationship between assigned break-and-burn age *b* and mean surface age μ_a is well described by the curve shown in the figure, which is:

$$\mu_a(b) = b \qquad \mathrm{b} \leq 12$$

$$\mu_a(b) = 26.58 \cdot \left(1 - \exp\left(-0.0614 \cdot (\mathrm{b} - 2.17)\right)\right) \qquad \mathrm{b} > 12$$

This is not precisely what we want, which is the relationship between *canonical* break-and–burn age *B* and mean surface age. The fitted curve is actually an errors-in-variables regression, but simulations show that the estimates are very close to the correct values, mainly because of the low variance of the break-and burn readings.

The data plotted in Fig. 2 consist of all paired readings from 1992 through 2002, representing all regulatory areas and both sexes, some read both ways not according to any experimental design but because of difficulty in assigning a surface age. So while large, it is a mixed and not entirely random sample. But the relationship between break-and-burn and surface ages is very similar for subsets of the data grouped by area or sex or reading type, so all of the data were pooled to compute a single working formula.

Variance of surface readings at a given break-and-burn age

The relationship between break-and-burn reading and the standard deviation of surface readings and is well described by the fitted logistic shown in Figure 3, which is:

$$\sigma_a(b) = 0.78 + 3.98 \Big/ \left(1 + \exp\left(-0.189 \cdot (b - 24.79)\right)\right)$$

This variance is inflated because the otoliths having a break-and-burn reading of, say, b' in fact consist of a mixture of canonical break-and-burn ages. Let B' denote that mixture, and let a and A denote assigned and canonical surface ages. The variance of a for a given b' can be represented as the sum of contributions from the component ages of B' by employing the handy rule that an unconditional variance is equal to the expectation of the conditional variance (over the component ages) plus the variance of the conditional expectation (over the component ages). Thus:

$$\sigma_a^2(b') = V(a \mid b') = \underset{B'}{E}\left[V(a \mid b', B)\right] + \underset{B'}{V}\left[E(a \mid b', B)\right]$$

For a given otolith the surface and break-and-burn readings are statistically independent, so in the first term in the sum $V(a \mid b', B) = V(a \mid B) = V(A \mid B) + V(a \mid A)$ and in the second term $E(a \mid b', B) = E(a \mid B) = E(A \mid B)$. Let $m(B) = E(A \mid B) \approx \mu_a(b)$ represent the relationship described above between canonical break-and-burn age B and mean canonical surface age A. With these substitutions:

$$\begin{aligned} V(a \mid b') &= \underset{B'}{E}\left[V(A \mid B) + V(a \mid A)\right] + \underset{B'}{V}\left[m(B)\right] \\ &\approx V(A \mid B = b') + V\left(a \mid A = m(b')\right) + m(b')^2 \cdot V(b \mid B = b') \end{aligned}$$

The form of the third term above relies on the fact that the distributions $f(B \mid b)$ and $f(b \mid B)$ are approximately equal, so $V(B \mid b) \approx V(b \mid B)$. This is just the variance of the break-and-burn readings about the canonical break-and-burn age. Here it is multiplied by the square of the slope of $m(B)$ at b' because locally $m(B) \approx m(b') + m'(b') \cdot (B - b')$ so $V(m(B)) \approx m'(b')^2 \cdot V(B \mid b')$. Among older fish this slope is low because surface age changes slowly with true age, so the third term contributes little to $V(a \mid b')$ at those ages.

The second and third terms can be computed from the known variances of surface and break-and-burn readings about their respective canonical ages. The first term is the variance of canonical surface age at a given canonical break-and-burn age. If all otoliths of a given canonical break-and-burn age belonged to a single canonical surface age, or a very narrow range, this term would be small. It is not. Through about age 15, it accounts for half or slightly less of the total variance, and beyond age 15 it increases steeply, by age 25 dwarfing the other variance components (Fig. 4).

Form of the distribution of surface ages at a given break-and-burn age

From about age 20 onward, where the variance and therefore the form of the distribution of surface ages $f(a|b')$ is dominated by $V(A|B)$, the distribution is very well approximated (Fig. 5) by a discrete version of the normal density:

$$f(a|b') \propto \exp\left(-\left(a-\mu_a(b')^2\right) \Big/ \left(2\cdot\sigma_a^2(b')\right)\right)$$

The standard deviation of this density is equal to the parameter σ_a for $\sigma_a \geq 0.6$; it is less than σ_a for smaller values.

Among younger fish, where other variance components are significant, the distributions are leptokurtic, having more data points at the mean and in the tails (and fewer in between) than a normal distribution with the same variance. This is clear in Fig. 5, where at low ages the normal distributions with parameters equal to the sample moments (thick gray lines) fail to reach the high observed frequencies of the modal (and true) ages. This shortfall can be prevented by using an ad hoc scaler to reduce the working value of σ_a to something less than the sample standard deviation. Specifically, if the sample standard deviation is multiplied by a scaler that increases linearly from 0.5 at age 5 to 1.0 at age 20, the normal approximation is adequate for the younger ages as well. The black lines in Fig. 4 are predicted frequencies based on the discrete normal distributions with means and standard deviations taken from the fitted curves reported above, except that at younger ages σ_a is calculated by scaling down the fitted standard deviation.

Discussion and conclusions

Predicting the distribution of break-and-burn readings of fish of a given true age B is straightforward because by hypothesis that is also the canonical break-and-burn age, so the readings can be expected to follow the simple geometric distribution described above and detailed in the Appendix.

Surface age readings are more complicated because their distribution depends on both the distribution of individual surface readings about the canonical surface age, $f(a|A)$, and the distribution of canonical surface age at a given true age or, equivalently, canonical break-and-burn age, $f(A|B)$. Beyond age 15 or 20, this source of variation is quite large. The canonical surface age of an otolith is the number of surface-countable annuli as defined in the protocol, and age readers can make that determination with a high degree of consistency. But among older otoliths of the same canonical break-and-burn age there is obviously a great deal of variation in the number of surface-countable annuli, which is not surprising.

The distribution $f(A|B)$ could be estimated by modeling, and surface readings of fish of each true age B could then be predicted in two steps by predicting the canonical surface age distribution and then the surface age reading distribution. A simpler alternative is to predict the distribution of break-and-burn readings $f(b|B)$ for all ages to obtain the overall marginal distribution of predicted break-and-burn readings and then use the simple model of $f(a|b)$ developed above to predict the corresponding distributions of surface readings. This approach has the attraction that both $f(b|B)$ and $f(a|b)$ can be (and have been) estimated directly from the available data. The smearing procedure is spelled out in the Appendix.

References

Clark, W.G. In review. Nonparametric estimates of age misclassification from paired readings.

Piner, K.R., and S. Wischniowski. In review. Description of a Pacific halibut chronology of bomb radiocarbon from 1944-1981 and a validation of ageing methods.

Appendix. Recipes.

This section is intended as a working reference for predicting the distribution of break-and-burn and surface readings of otoliths of a given true age *B*; it contains no new material.

Distribution of break-and-burn readings

Let $v = b - B$, the signed deviation of a single reading from the true age. The distribution of v is $f(0) = p$ and $f(v \neq 0) = p \cdot q^{|v|} / 2$ where $q = 1 - p$ and the parameter $p = f(0)$ is a function of the variance of break-and-burn readings at age *B*:

$$p = \frac{-3 + \sqrt{1 + 8 \cdot \sigma_v^2}}{2 \cdot (\sigma_v^2 - 1)} \text{ if } \sigma_v^2 \neq 1 \text{ else } 2/3$$

$$\sigma_v^2(B) = \left(1.28 \cdot \left(1 - \exp\left(-0.100 \cdot (B - 2.93)\right)\right)\right)^2$$

Distribution of surface readings

The first step is to generate the distribution of predicted readings (assigned ages) *b* for fish of all true ages, including *B*, as described above. For each assigned age b', the mean μ_a of predicted surface readings is:

$$\mu_a(b') = b' \qquad b' \leq 12$$
$$\mu_a(b') = 26.58 \cdot \left(1 - \exp\left(-0.0614 \cdot (b' - 2.17)\right)\right) \qquad b' > 12$$

The actual standard deviation of surface readings is:

$$\sigma_a(b') = 0.78 + 3.98 / \left(1 + \exp\left(-0.189 \cdot (b' - 24.79)\right)\right)$$

but the computations use a working value $\delta_a = c \cdot \sigma_a$ that is scaled down at the lower ages. The value of the scaler *c* is:

$$c = 0.5 \qquad b' < 5$$
$$c = 0.5 + 0.5 \cdot (b' - 5)/15 \qquad 5 \leq b' \leq 20$$
$$c = 1 \qquad b' > 20$$

Let $d(a) = \exp\left(-(a - \mu_a)^2 / (2 \cdot \delta_a^2)\right)$, a discrete form of part of the normal density function. Note the last term is δ_a^2 not just δ_a. Normalizing the values of $d(a)$ finally gives the density of surface readings at break-and-burn age b': $f(a) = d(a) / \sum_a d(a)$.

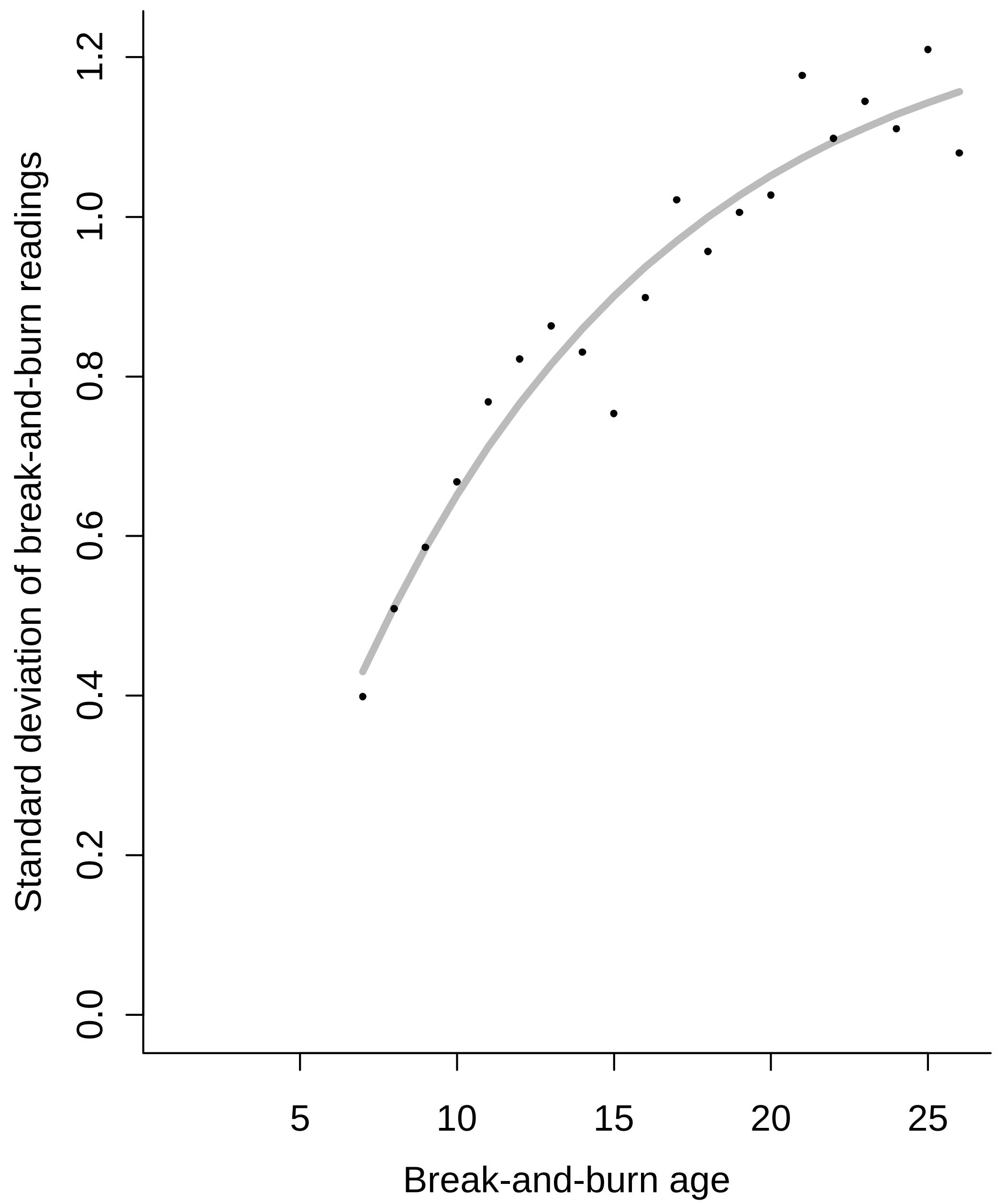

Figure 1. Standard deviation of break-and-burn readings as function of age. The abscissa is the mean assigned age in paired readings. The ordinate is the root of half the variance of the signed difference of paired readings. The gray line is a fitted curve; see text.

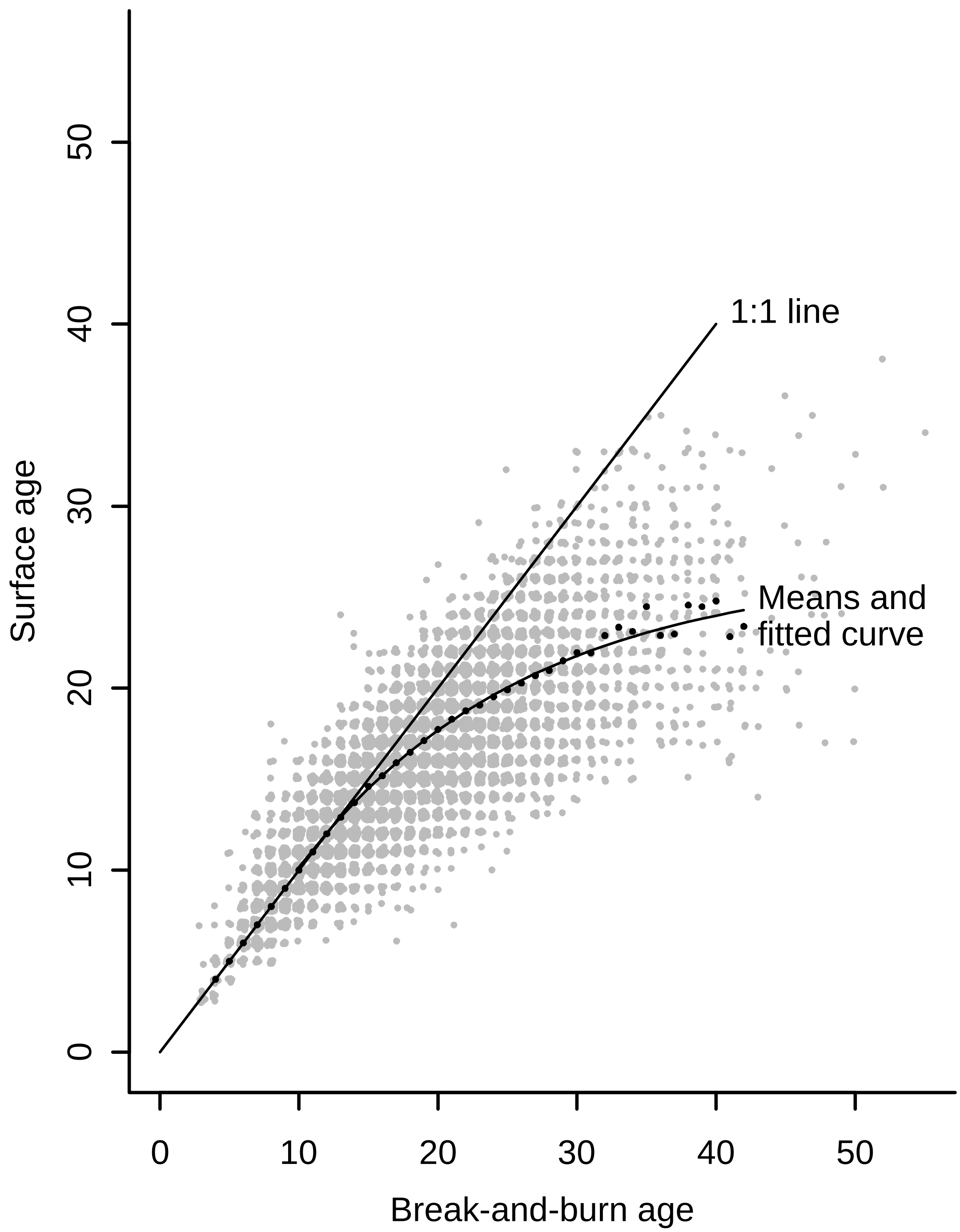

Figure 2. Surface age reading plotted against break-and-burn age reading of the same 60,000 otoliths. The gray masses are the raw data points (jittered). The black points are the mean surface age at each break-and-burn age.

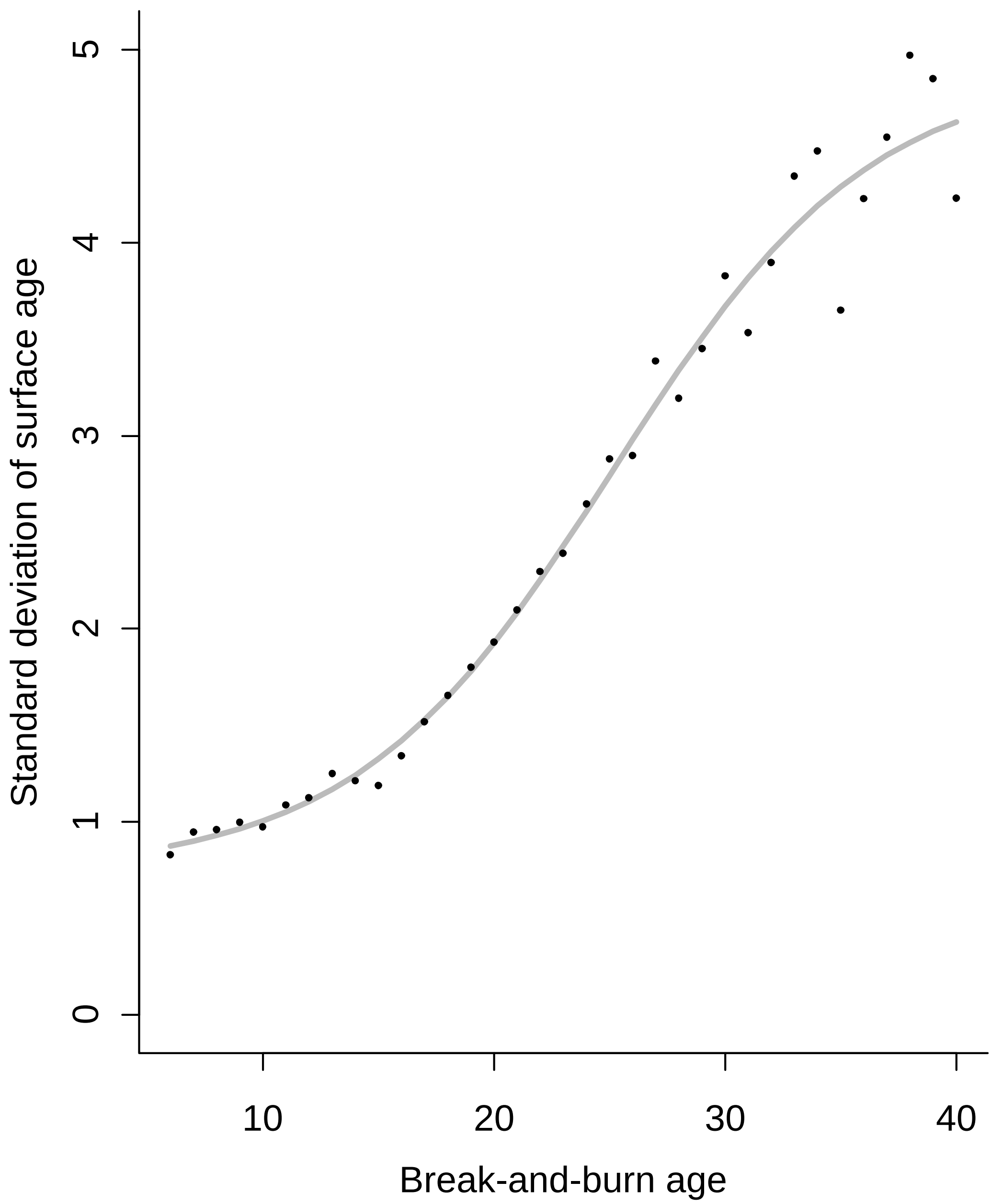

Figure 3. Standard deviation of surface readings plotted against break-and-burn readings (points), and a fitted logistic.

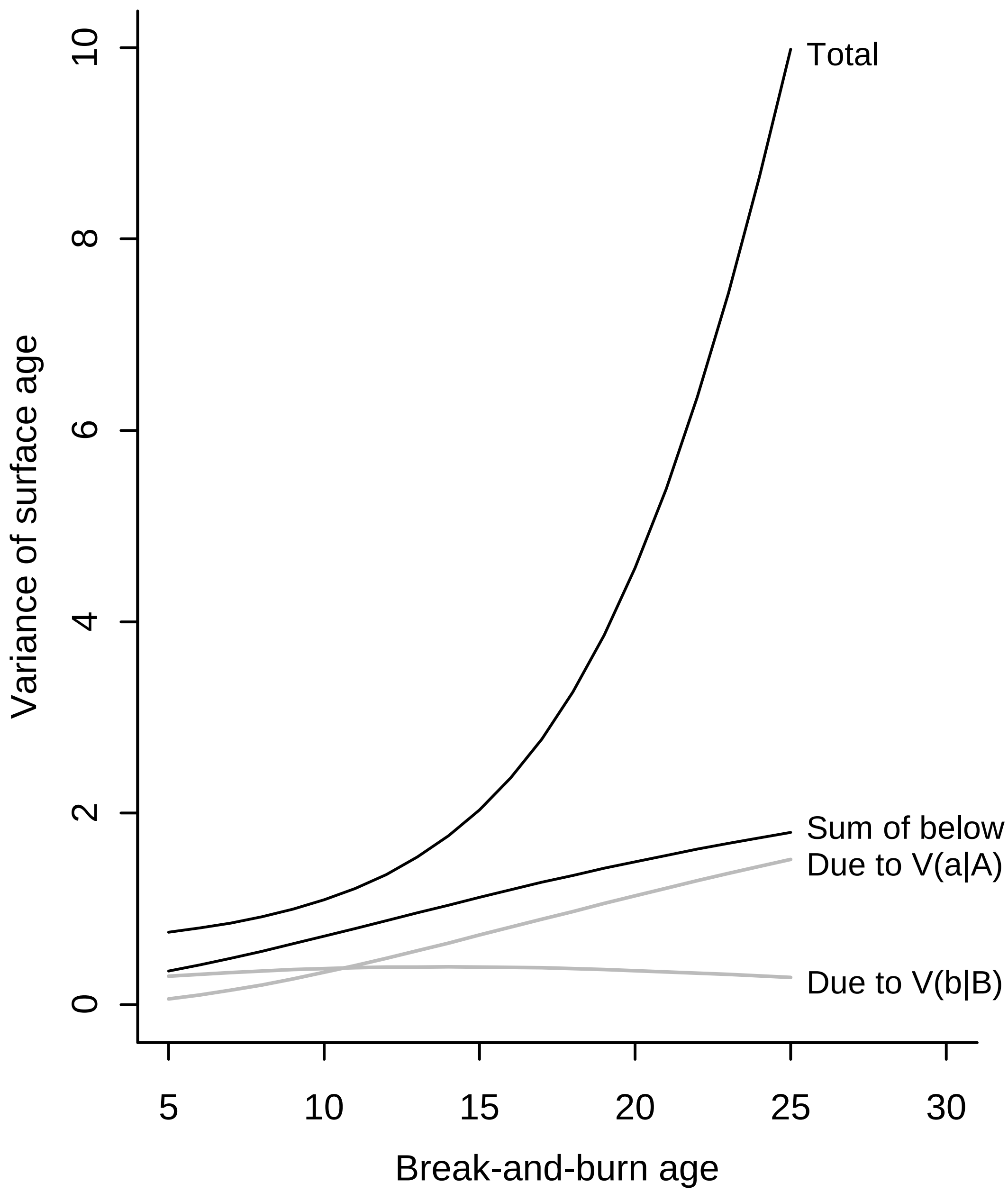

Figure 4. Components of the variance of surface readings at a given break-and-burn reading. V(a|A) is the variance of surface readings about the canonical surface age. V(b|B) is the same for break-and-burn readings, but only a fraction enters the total variance; see text. The difference between the two upper lines is due to V(A|B), the variance of canonical surface age at a given canonical break-and-burn age.

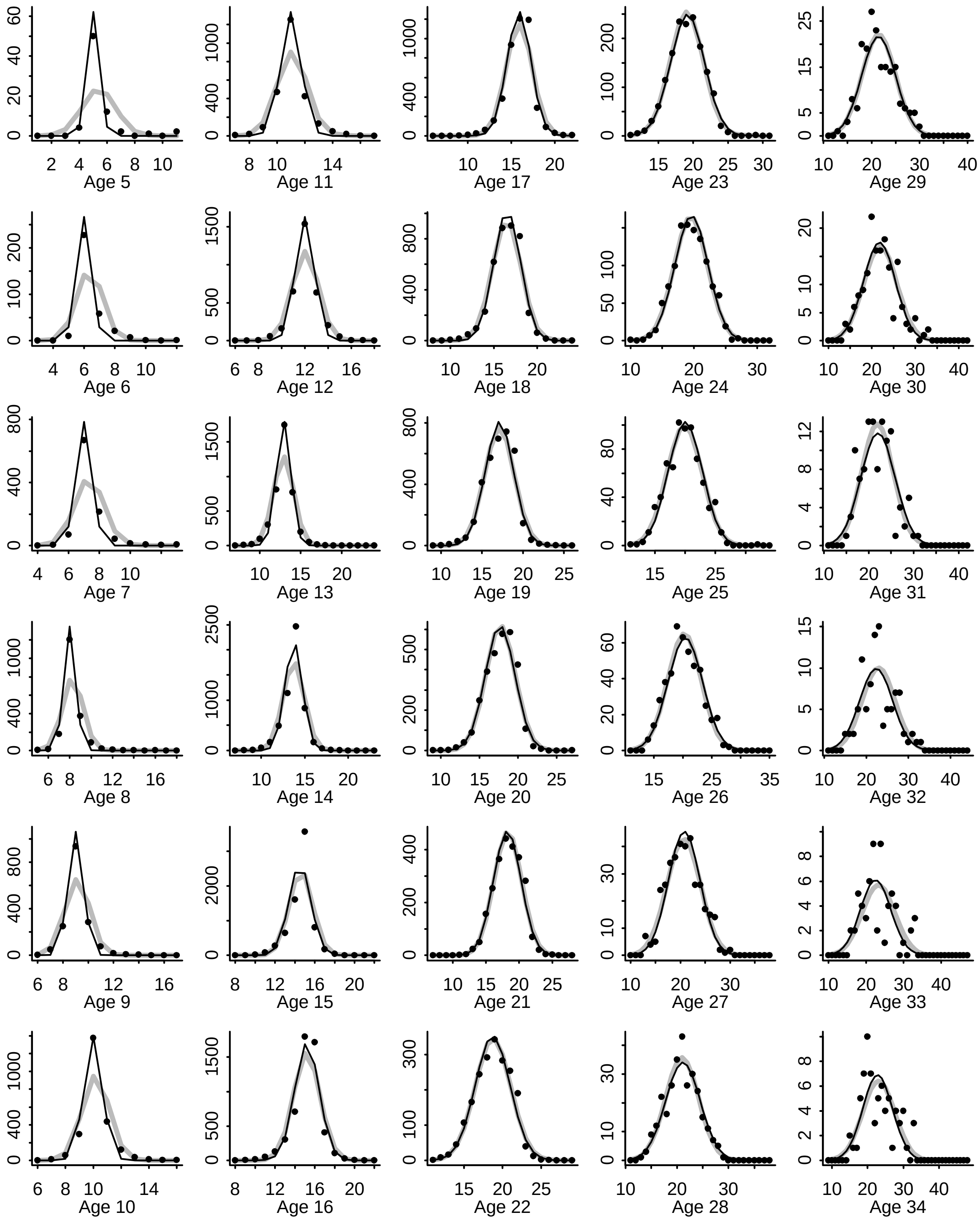

Figure 5. Observed and predicted frequencies (number of otoliths) of surface ages grouped by *assigned* break-and-burn age. The gray line in each plot is a discrete version of the normal density with parameters equal to the sample moments. The black line is a normal density with parameters calculated from the fitted curves reported in the text.